

where the following denotation is used:

$$\begin{aligned}
& \vec{\chi}_{12}(\vec{r}_0, \vec{\rho}_C, M, \dot{\omega}_1, \dot{\omega}_2, \omega_1, \omega_2, \vec{n}_1, \vec{n}_2) \\
&= \dot{\omega}_1 [\vec{\rho}_C, [\vec{n}_1, \vec{r}_0]] M + \omega_1^2 [\vec{n}_1, [\vec{\rho}_C, [\vec{n}_1, \vec{r}_0]]] M \\
&+ \dot{\omega}_1 \left[\vec{r}_0, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} \right] + \dot{\omega}_2 \left[\vec{r}_0, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right] + \omega_1^2 \left[\vec{n}_1, \left[\vec{r}_0, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} \right] \right] + \omega_2^2 \left[\vec{n}_2, \left[\vec{r}_0, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right] \right] \\
&+ \omega_1^2 \vec{n}_1 \left(\vec{r}_0, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} \right) M + \omega_2^2 \left[\vec{r}_0, \left[\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right] \right] - \omega_1 \omega_2 \left[[\vec{n}_1, \vec{r}_0], \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right] M \quad (3.8) \\
&+ \omega_1^2 M \vec{n}_1 (\vec{\rho}_C, [\vec{n}_1, \vec{r}_0]) + \omega_1 \omega_2 \left[[\vec{n}_1, \vec{r}_0], \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right] M \\
&+ \omega_2 \omega_1 \left[\vec{r}_0, \left[\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right] \right] + \omega_1 \omega_2 \left[\vec{r}_0, \left[\vec{n}_2, \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} \right] \right] \\
&+ \omega_1 \omega_2 (\vec{\rho}_C, \vec{n}_1) [\vec{r}_0, \vec{n}_2] M - \omega_1 \omega_2 (\vec{\rho}_C, \vec{n}_2) [\vec{r}_0, \vec{n}_1] M.
\end{aligned}$$

4. Vector Rotators of Rigid Body Coupled Rotations around Two Axes without Intersection

We can see that in previous expression (3.5) for derivative of linear momentum the following three vectors are introduced:

$$\begin{aligned}
& \vec{\mathfrak{R}}_{01} = \dot{\omega}_1 \vec{u}_{01} + \omega_1^2 \vec{v}_{01}, \\
& \vec{\mathfrak{R}}_{01} = \dot{\omega}_1 \left[\vec{n}_1, \frac{\vec{r}_0}{r_0} \right] + \omega_1^2 \left[\vec{n}_1, \left[\vec{n}_1, \frac{\vec{r}_0}{r_0} \right] \right], \\
& \vec{\mathfrak{R}}_{011} = \dot{\omega}_1 \vec{u}_{011} + \omega_1^2 \vec{v}_{011}, \\
& \vec{\mathfrak{R}}_{011} = \dot{\omega}_1 \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}}{\left| \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} \right|} + \omega_1^2 \left[\vec{n}_1, \frac{\vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)}}{\left| \vec{\mathfrak{S}}_{\vec{n}_1}^{(O_2)} \right|} \right] = \dot{\omega}_1 \frac{[\vec{n}_1, \vec{\rho}_C]}{\left| [\vec{n}_1, \vec{\rho}_C] \right|} + \omega_1^2 \frac{[\vec{n}_1, [\vec{n}_1, \vec{\rho}_C]]}{\left| [\vec{n}_1, \vec{\rho}_C] \right|}, \quad (4.1) \\
& \vec{\mathfrak{R}}_{022} = \dot{\omega}_1 \vec{u}_{022} + \omega_1^2 \vec{v}_{022}, \\
& \vec{\mathfrak{R}}_{022} = \dot{\omega}_2 \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}}{\left| \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right|} + \omega_2^2 \left[\vec{n}_2, \frac{\vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)}}{\left| \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right|} \right] = \dot{\omega}_2 \frac{[\vec{n}_2, \vec{\rho}_C]}{\left| [\vec{n}_2, \vec{\rho}_C] \right|} + \omega_2^2 \frac{[\vec{n}_2, [\vec{n}_2, \vec{\rho}_C]]}{\left| [\vec{n}_2, \vec{\rho}_C] \right|}.
\end{aligned}$$

The first two vector rotators $\vec{\mathfrak{R}}_{01}$ and $\vec{\mathfrak{R}}_{011}$ are orthogonal to the direction of the first fixed axis and third vector rotator $\vec{\mathfrak{R}}_{022}$ is orthogonal to the self rotation axis. But, first vector rotator $\vec{\mathfrak{R}}_{01}$ is coupled for pole O_1 on the fixed axis and second and third vector rotators, $\vec{\mathfrak{R}}_{011}$ and $\vec{\mathfrak{R}}_{022}$, are coupled for the pole O_2 at self rotation axis and for corresponding direction oriented by directions of component angular velocities of coupled rotations. Intensity of two

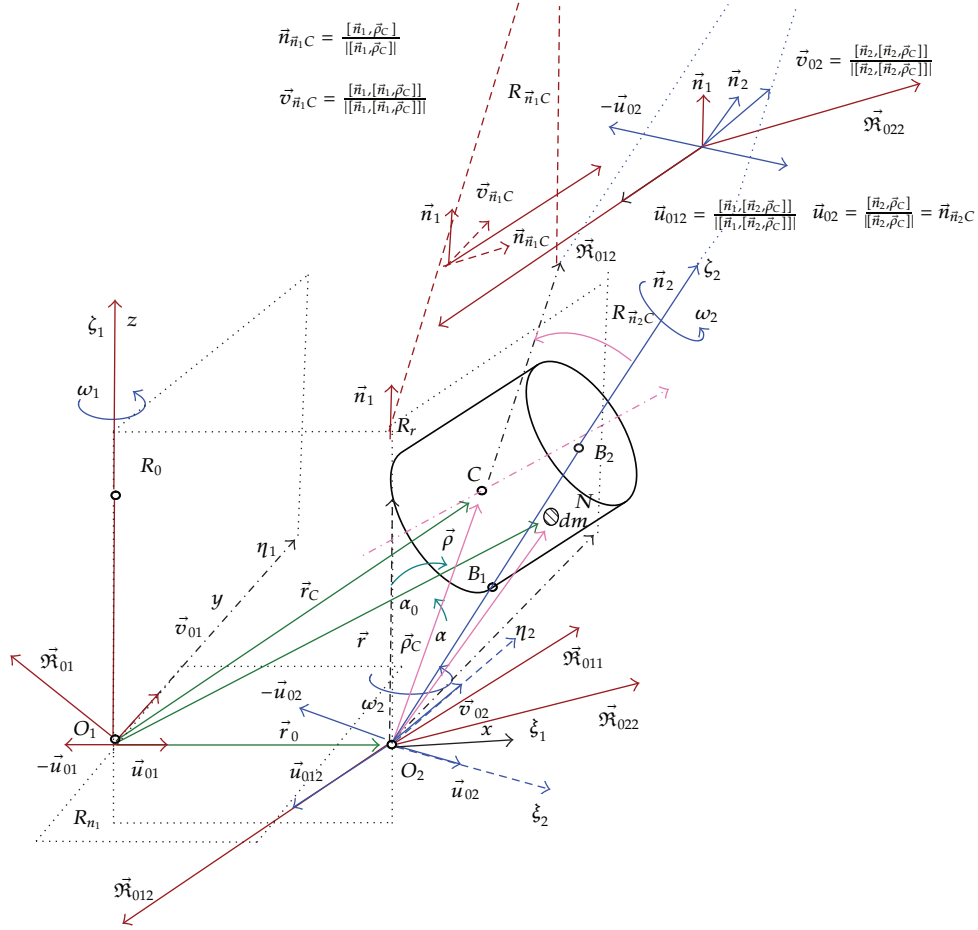


Figure 1: Arbitrary position of rigid body coupled rotations around two axes without intersection. System is with two degrees of mobility (two freedom or one degree of freedom and one rheonomic constraint) where φ_1 and φ_2 are generalized coordinates fixed coordinate system and two moveable coordinate systems $O_1 \xi_1 \eta_1 \zeta_1 = O_1 \xi_1 \eta_1 z$ and $O_2 \xi_2 \eta_2 \zeta_2 = O_2 \xi_2 \eta_2 z_2$ that are rotating with component angular velocities of rigid body coupled rotations: independent generalized (/or rheonomic) coordinates are φ_1 coordinate of precession rotation and φ_2 coordinate around self rotation axis. Vector rotators $\vec{\mathfrak{R}}_{01}$, $\vec{\mathfrak{R}}_{011}$, and $\vec{\mathfrak{R}}_{022}$ are presented.

first rotators is equal and is expressed by angular velocity and angular acceleration of the first component rotation, and intensity of third vector rotators is expressed by angular velocity and angular acceleration of the second component rotation, and are in the following forms:

$$\mathfrak{R}_{01} = \mathfrak{R}_{011} = \sqrt{\dot{\omega}_1^2 + \omega_1^4}, \quad \mathfrak{R}_{022} = \sqrt{\dot{\omega}_2^2 + \omega_2^4}. \quad (4.2)$$

Lets introduce notation γ_{01} , γ_{011} , and γ_{022} denote difference between corresponding component angles of rotation φ_1 and φ_2 of the rigid body component rotations and

corresponding absolute angles of pure kinematics vector rotators about axes oriented by unit vectors \vec{n}_1 and \vec{n}_2 . These angles are determined by the following relations:

$$\gamma_{01} = \gamma_{011} = \arctan \frac{\dot{\varphi}_1^2}{\ddot{\varphi}_1}, \quad \gamma_{02} = \arctan \frac{\dot{\varphi}_2^2}{\ddot{\varphi}_2}. \quad (4.3)$$

Angular velocity of relative kinematics vectors rotators $\vec{\mathfrak{R}}_{01}$, $\vec{\mathfrak{R}}_{011}$, and $\vec{\mathfrak{R}}_{022}$ which rotate about corresponding axes in relation to the component angular velocities of the rigid body component rotations are

$$\dot{\gamma}_{01} = \dot{\gamma}_{011} = \frac{\dot{\varphi}_1(2\ddot{\varphi}_1 - \dot{\varphi}_1\ddot{\varphi}_1)}{\ddot{\varphi}_1^2 + \dot{\varphi}_1^4}, \quad \dot{\gamma}_{02} = \frac{\dot{\varphi}_2(2\ddot{\varphi}_2 - \dot{\varphi}_2\ddot{\varphi}_2)}{\ddot{\varphi}_2^2 + \dot{\varphi}_2^4}. \quad (4.4)$$

In Figure 1. Vector rotators $\vec{\mathfrak{R}}_{01}$, $\vec{\mathfrak{R}}_{011}$, and $\vec{\mathfrak{R}}_{022}$ are presented.

Fourth vector rotator $\vec{\mathfrak{R}}_{012}$ is in the following vector form and with intensity \mathfrak{R}_{012} :

$$\vec{\mathfrak{R}}_{012} = 2\omega_1\omega_2 \frac{\left[\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right]}{\left| \left[\vec{n}_1, \vec{\mathfrak{S}}_{\vec{n}_2}^{(O_2)} \right] \right|} = 2\omega_1\omega_2 \frac{[\vec{n}_1, [\vec{n}_2, \vec{\rho}_C]]}{\left| [\vec{n}_1, [\vec{n}_2, \vec{\rho}_C]] \right|}, \quad \left| \vec{\mathfrak{R}}_{012} \right| = \mathfrak{R}_{012} = 2\omega_1\omega_2 \quad (4.5)$$

This vector rotator $\vec{\mathfrak{R}}_{012}$ depends on both components of coupled rotations.

We can see that in previous vector expression (3.6) or (3.7) for derivative of angular momentum are introduced following two vectors rotators: $\vec{\mathfrak{R}}_1 = \dot{\omega}_1\vec{u}_1 + \omega_1^2\vec{v}_1$ and $\vec{\mathfrak{R}}_2 = \dot{\omega}_2\vec{u}_2 + \omega_2^2\vec{v}_2$ in the following vector form:

$$\begin{aligned} \vec{\mathfrak{R}}_1 &= \dot{\omega}_1 \frac{\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)} \right|} + \omega_1^2 \left[\vec{n}_1, \frac{\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)} \right|} \right] = \dot{\omega}_1\vec{u}_1 + \omega_1^2\vec{v}_1, \\ \vec{\mathfrak{R}}_2 &= \dot{\omega}_2 \frac{\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)} \right|} + \omega_2^2 \left[\vec{n}_2, \frac{\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)} \right|} \right] = \dot{\omega}_2\vec{u}_2 + \omega_2^2\vec{v}_2. \end{aligned} \quad (4.6)$$

The first $\vec{\mathfrak{R}}_1$ is orthogonal to the fixed axis oriented by unit vector \vec{n}_1 and second $\vec{\mathfrak{R}}_2$ is orthogonal to the self rotation axis oriented by unit vector \vec{n}_2 . Intensity of first rotator $\vec{\mathfrak{R}}_1$ is equal to intensity of previous defined rotator \mathfrak{R}_{01} and intensity of second rotator $\vec{\mathfrak{R}}_2$ is equal to intensity of previous defined rotator \mathfrak{R}_{022} defined by expressions (3.7). Their intensities are

$$\mathfrak{R}_1 = \sqrt{\dot{\omega}_1^2 + \omega_1^4}, \quad \mathfrak{R}_2 = \sqrt{\dot{\omega}_2^2 + \omega_2^4}. \quad (4.7)$$

In Figure 2 vector rotators $\vec{\mathfrak{R}}_1$ (in Figure 2(a)) and $\vec{\mathfrak{R}}_2$ (in Figure 2(b)) in relations to corresponding mass moment vectors $\vec{\mathfrak{J}}_{\vec{n}_1}^{(O_2)}$ and $\vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)}$, and their corresponding deviational components $\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}$ and $\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}$ as well as to corresponding deviational planes are presented.

Vector rotators $\vec{\mathfrak{R}}_1$ and $\vec{\mathfrak{R}}_2$ are pure kinematical vectors first presented in [20, 21] as a function on angular velocity and angular acceleration in a form $\vec{\mathfrak{R}} = \dot{\varphi}\vec{u} + \dot{\varphi}^2\vec{w} = \mathfrak{R}\vec{\mathfrak{R}}_0$. Also from Section 3.3 expressions (3.5) and (3.6) or (3.7) for derivatives for linear and angular momentum contain members with in tree types of different pure kinematical vectors rotators which rotate around first and second axis in corresponding directions of coupled rotation components, but with pole in O_1 or in O_2 . These vector rotators are possible to separate by following criteria: (1) intensity of vector rotator is expressed by angular velocity ω_1 and angular acceleration $\dot{\omega}_1$ in the form $\mathfrak{R}_1 = \sqrt{\dot{\omega}_1^2 + \omega_1^4}$ or angular velocity ω_2 and angular acceleration $\dot{\omega}_2$ in the form and $\mathfrak{R}_2 = \sqrt{\dot{\omega}_2^2 + \omega_2^4}$; (2) intensity of the vector rotators is expressed by both angular velocity components ω_1 and ω_2 , and no contain angular accelerations $\dot{\omega}_1$ and $\dot{\omega}_2$; (3) vector rotators are coupled by pole in O_1 or in O_2 ; (4) type of angular velocities components of vector rotators.

Rotators from first set are rotated around through pole O_2 axis in direction of first component rotation angular velocity and depend of angular velocity ω_1 and angular acceleration $\dot{\omega}_1$. There are two vectors of such type and all trees have equal intensity. Rotators from second set are rotated around axis in direction of second component rotation and depend of angular velocity ω_2 and angular acceleration $\dot{\omega}_2$. There are two vectors of such type and they have equal intensity.

Let us introduce notation, γ_1 and γ_2 denote difference between corresponding component angles of rotation φ_1 and φ_2 of the rigid body component rotations and corresponding absolute angles of pure kinematics vector rotators about axes oriented by unit vectors \vec{n}_1 and \vec{n}_2 through pole O_2 . These angles are determined by following relations:

$$\gamma_1 = \arctan \frac{\dot{\varphi}_1^2}{\varphi_1^2}, \quad \gamma_2 = \arctan \frac{\dot{\varphi}_2^2}{\varphi_2^2}. \quad (4.8)$$

Angular velocity of relative kinematics vectors rotators $\vec{\mathfrak{R}}_1$ and $\vec{\mathfrak{R}}_2$ which rotate about axes in corresponding directions in relation to the component angular velocities of the rigid body component rotations through pole O_2 are

$$\dot{\gamma}_1 = \frac{\dot{\varphi}_1(2\ddot{\varphi}_1^2 - \dot{\varphi}_1\ddot{\varphi}_1)}{\ddot{\varphi}_1^2 + \dot{\varphi}_1^4}, \quad \dot{\gamma}_2 = \frac{\dot{\varphi}_2(2\ddot{\varphi}_2^2 - \dot{\varphi}_2\ddot{\varphi}_2)}{\ddot{\varphi}_2^2 + \dot{\varphi}_2^4}. \quad (4.9)$$

Also, it is possible to separate a few numbers of rotators and between the following:

$$\vec{\mathfrak{R}}_{12} = 2\omega_1\omega_2 \frac{\left[\vec{n}_1, \vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)} \right]}{\left| \left[\vec{n}_1, \vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)} \right] \right|} = 2\omega_1\omega_2 \vec{u}_{12}, \quad (4.10)$$

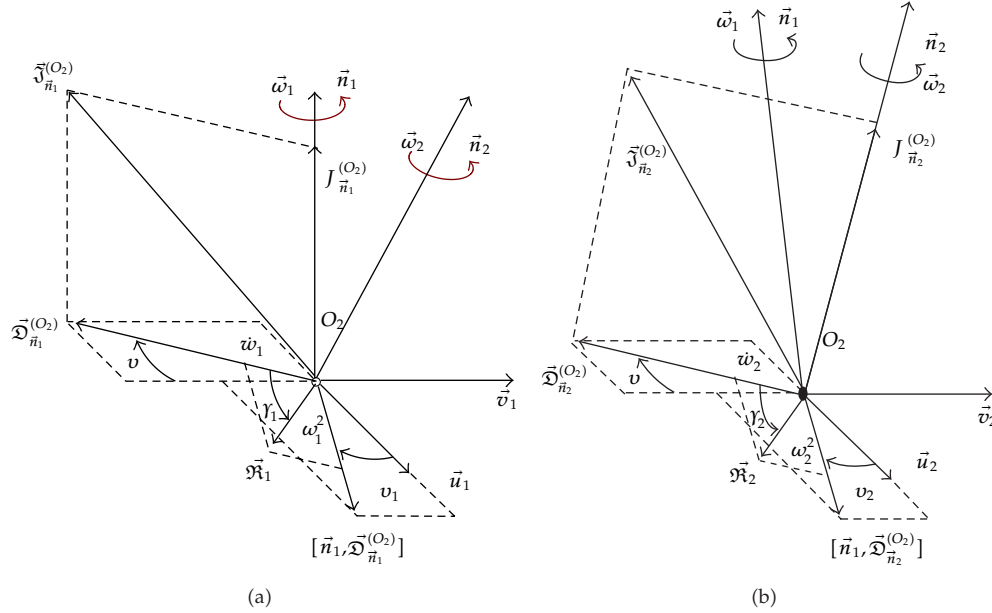


Figure 2: Vector rotators $\vec{\mathfrak{R}}_1$ (a) and $\vec{\mathfrak{R}}_2$ (b) in relations to corresponding mass moment vectors $\vec{J}_{\vec{n}_1}^{(O_2)}$ and $\vec{J}_{\vec{n}_2}^{(O_2)}$, and their corresponding deviativonal components $\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}$ and $\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}$ as well as to corresponding deviativonal planes.

where $\vec{u}_{12} = [\vec{n}_1, \vec{J}_{\vec{n}_2}^{(O_2)}] / |[\vec{n}_1, \vec{J}_{\vec{n}_2}^{(O_2)}]|$ unit vector orthogonal to the axis oriented by unit vector \vec{n}_1 and mass moment vector $\vec{J}_{\vec{n}_2}^{(O_2)}$ for the axis oriented by unit vector \vec{n}_2 through pole O_2 , and intensity equal $\mathfrak{R}_{12} = 2\omega_1\omega_2$ twice multiplication of product of intensities of component angular velocities ω_1 and ω_2 of rigid body coupled rotations around axes without intersection.

5. Vector Rotators of Rigid Body-Disk Dynamics with Coupled Rotations around Two Orthogonal Axes without Intersection

Let us consider vector rotators for the special case when rigid body-disk rotate around two orthogonal axes without intersection.

Vector of relative mass center position $\vec{\rho}_C$ in relation to the pole O_2 and self rotation axis oriented by unit vector \vec{n}_2 , we can express in the movable coordinate systems with axes oriented by basic unit vectors: \vec{n}_2, \vec{u}_{02} and \vec{v}_{02} which rotate around self rotation axis with angular velocity ω_2 in the form $\vec{\rho}_C = \rho_C(\cos \beta \vec{n}_2 + \sin \beta \vec{u}_{02})$, as well as by basic unit vectors $\vec{u}_{01}, \vec{v}_{01}$ and \vec{n}_1 which rotate around fixed axis oriented by unit vector \vec{n}_1 with angular velocity ω_1 in the following form: $\vec{\rho}_C = \rho_C(\cos \beta \vec{u}_{01} - \sin \beta \cos \varphi_2 \vec{v}_{01} + \sin \beta \sin \varphi_2 \vec{n}_1)$. β is angle between mass center vector position $\vec{\rho}_C$ and self rotation axis oriented by unit vector \vec{n}_2 . Vector of the orthogonal distance between orthogonal axes without intersection is $\vec{r}_0 = -r_0 \vec{v}_{01}$.

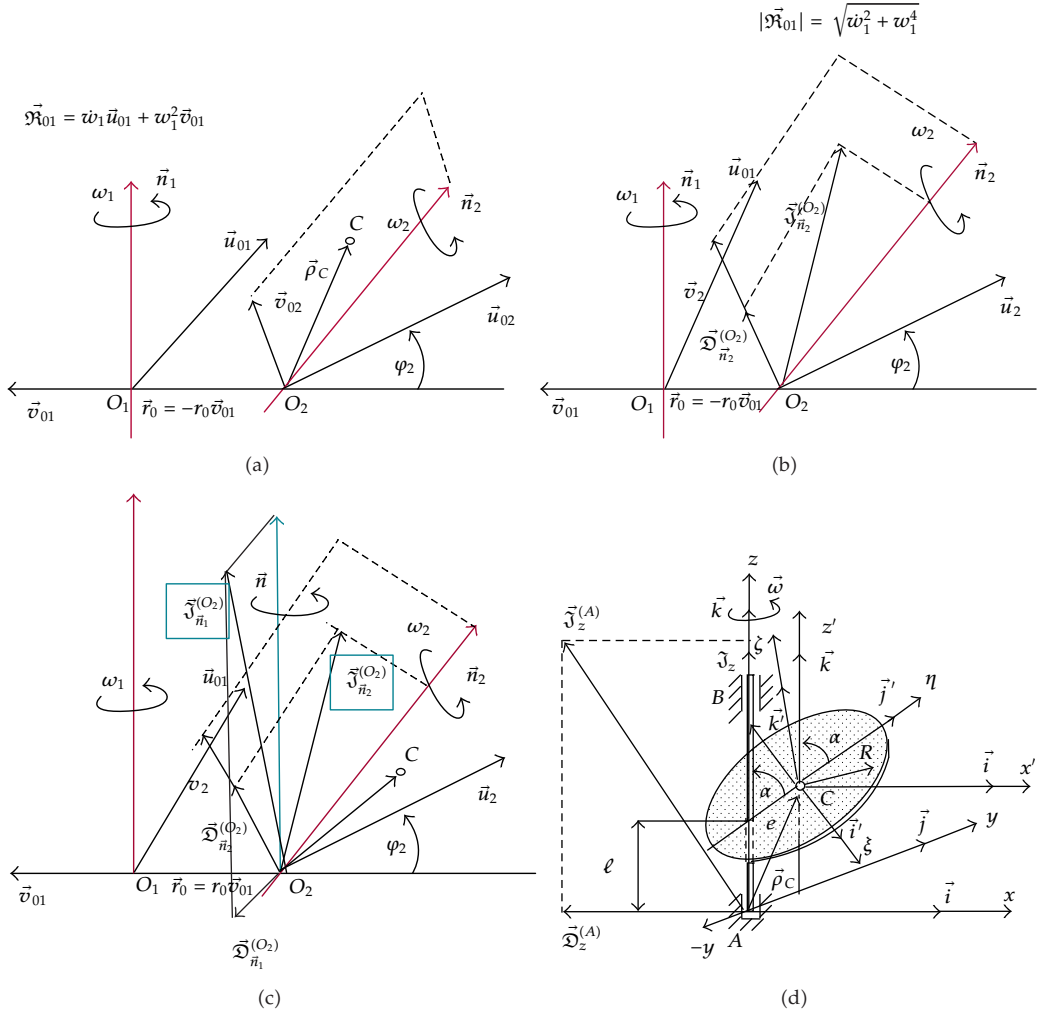


Figure 3: Schematic presentation of deviational planes and component directions of the vector rotators of rigid body-disk dynamics with coupled rotation around two orthogonal axes without intersection. (a) Deviation plane containing body mass center C , vector of relative mass center position $\vec{\rho}_C$ in relation to the pole O_2 , and self rotation axis oriented by unit vector \vec{n}_2 . (b) Deviation plane containing self rotation axis oriented by unit vector \vec{n}_2 and body mass inertia moment vector $\vec{J}_{\vec{n}_2}^{(O_2)}$ and its deviational component vector of mass deviational moment $\vec{D}_{\vec{n}_2}^{(O_2)}$ for self rotation axis and pole O_2 . (c) Two deviational planes through pole O_2 : deviation plane containing self rotation axis oriented by unit vector \vec{n}_2 and body mass inertia moment vector $\vec{J}_{\vec{n}_2}^{(O_2)}$ and its deviational component vector of mass deviational moment $\vec{D}_{\vec{n}_2}^{(O_2)}$ for self rotation axis and pole O_2 and deviation plane containing axis parallel to fixed axis oriented by unit vector \vec{n}_1 and body mass inertia moment vector $\vec{J}_{\vec{n}_1}^{(O_2)}$ and its deviational component vector of mass deviational moment $\vec{D}_{\vec{n}_1}^{(O_2)}$ for axis oriented by unit vector \vec{n}_1 and through pole O_2 . (d) Schematic presentation of the rigid body-disk skew and eccentrically positioned on the self rotation axis with corresponding mass moment vectors and deviation plane as a detail of the rigid body-disk coupled rotation around two orthogonal axes without intersection.

For this case unit vectors \vec{n}_1 and \vec{n}_2 are orthogonal, and after taking into account this orthogonality and corresponding formulas (4.1), (4.5), (4.6), and (4.10) for vector rotators we obtain the following vector expressions:

$$\begin{aligned}
\vec{\mathfrak{R}}_{011} &= \frac{\langle \vec{v}_{01}(\dot{\omega}_1 \cos \beta - \omega_1^2 \sin \beta \sin \varphi_2) - \vec{u}_{01}(\dot{\omega}_1 \sin \beta \sin \varphi_2 + \omega_1^2 \cos \beta) \rangle}{\sqrt{\cos^2 \beta + \sin^2 \beta \sin^2 \varphi_2}} & |\vec{\mathfrak{R}}_{011}| &= \sqrt{\dot{\omega}_1^2 + \omega_1^4} \\
\vec{\mathfrak{R}}_{022} &= \dot{\omega}_2 \vec{v}_{02} - \omega_1^2 \vec{u}_{02} & |\vec{\mathfrak{R}}_{022}| &= \sqrt{\dot{\omega}_2^2 + \omega_2^4} \\
\vec{\mathfrak{R}}_{012} &= -2\omega_1 \omega_2 \vec{u}_{01} & |\vec{\mathfrak{R}}_{012}| &= \mathfrak{R}_{012} = 2\omega_1 \omega_2 \\
\vec{\mathfrak{R}}_1 &= -\frac{\vec{u}_{01} \langle \dot{\omega}_1 \cos \beta + \omega_1^2 \sin \beta \cos \varphi_2 \rangle + \vec{v}_{01} \langle -\dot{\omega}_1 \sin \beta \cos \varphi_2 + \omega_1^2 \cos \beta \rangle}{\sqrt{\cos^2 \beta + \sin^2 \beta \cos^2 \varphi_2}} & |\vec{\mathfrak{R}}_1| &= \sqrt{\dot{\omega}_1^2 + \omega_1^4} \\
\vec{\mathfrak{R}}_2 &= \dot{\omega}_2 \vec{v}_{02} - \omega_1^2 \vec{u}_{02} & |\vec{\mathfrak{R}}_2| &= \sqrt{\dot{\omega}_2^2 + \omega_2^4} \\
\vec{\mathfrak{R}}_{12} &= 2\omega_1 \omega_2 \frac{\left[\vec{n}_1, \vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)} \right]}{\left| \left[\vec{n}_1, \vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)} \right] \right|} = 2\omega_1 \omega_2 \vec{u}_{12} & |\vec{\mathfrak{R}}_{12}| &= \mathfrak{R}_{12} = 2\omega_1 \omega_2.
\end{aligned} \tag{5.1}$$

Previous expressions for vectors rotators are derived with supposition that rigid body is disk and that unit vectors in different deviation planes are:

$$\begin{aligned}
\frac{\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)} \right|} &= -\frac{\langle \cos \beta \vec{u}_{01} - \sin \beta \cos \varphi_2 \vec{v}_{01} \rangle}{\sqrt{\cos^2 \beta + \sin^2 \beta \cos^2 \varphi_2}} & \left[\vec{n}_1, \frac{\vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_1}^{(O_2)} \right|} \right] &= -\frac{\cos \beta \vec{v}_{01} + \sin \beta \cos \varphi_2 \vec{u}_{01}}{\sqrt{\cos^2 \beta + \sin^2 \beta \cos^2 \varphi_2}} \\
\vec{v}_{02} = \vec{u}_2 &= \frac{\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)} \right|} & \vec{u}_{02} = \vec{v}_2 &= \left[\vec{n}_2, \frac{\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}}{\left| \vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)} \right|} \right].
\end{aligned} \tag{5.2}$$

In Figure 3. four schematic presentations of deviational planes and component directions of the vector rotators of rigid body-disk dynamics with coupled rotation around two orthogonal axes without intersection are presented. In Figure 3(a) deviation plane containing body mass center C , vector of relative mass center position $\vec{\rho}_C$ in relation to the pole O_2 and self rotation axis oriented by unit vector \vec{n}_2 is visible. In Figure 3(b) deviation plane containing self rotation axis oriented by unit vector \vec{n}_2 and body mass inertia moment vector $\vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)}$ and its deviational component vector of mass deviational moment $\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}$ for self rotation axis and pole O_2 is visible. In Figure 3(c) two deviational planes through pole O_2 : deviation plane containing self rotation axis oriented by unit vector \vec{n}_2 and body mass inertia moment vector $\vec{\mathfrak{J}}_{\vec{n}_2}^{(O_2)}$ and its deviational component vector of mass deviational moment $\vec{\mathfrak{D}}_{\vec{n}_2}^{(O_2)}$

for self rotation axis and pole O_2 and deviation plane containing axis parallel to fixed axis oriented by unit vector \vec{n}_1 and body mass inertia moment vector $\vec{J}_{\vec{n}_1}^{(O_2)}$ and its deviational component vector of mass deviational moment $\vec{D}_{\vec{n}_1}^{(O_2)}$ for axis oriented by unit vector \vec{n}_1 and through pole O_2 are visible. In Figure 3(d) schematic presentation of the rigid body-disk skew and eccentrically positioned on the self rotation axis with corresponding mass moment vectors and deviation plane as a detail of the rigid body-disk coupled rotation around two orthogonal axes without intersection is visible. In all form of the parts in Figure 3. the component directions of the vector rotators components are visible.

By use derived vector expressions of the vector rotators we can obtain some angles between corresponding vector rotator and basic vectors of corresponding movable coordinate systems coupled with corresponding compounding axis of component coupled rotations in the following form:

$$\begin{aligned} \operatorname{tg}\gamma_1 &= \frac{\dot{\omega}_1}{\omega_1^2} & \operatorname{tg}\gamma_{011} &= \frac{\dot{\omega}_1}{\omega_1^2} = \operatorname{tg}\gamma_1 \\ \operatorname{tg}\tilde{\gamma}_1 &= \frac{1 - (\dot{\omega}_1/\omega_1^2)\operatorname{tg}\beta \cos \varphi_2}{(\dot{\omega}_1/\omega_1^2) + \operatorname{tg}\beta \cos \varphi_2} & \text{or in the form} & \operatorname{tg}\tilde{\gamma}_1 = \frac{1 - \operatorname{tg}\gamma_1 \operatorname{tg}\beta \cos \varphi_2}{\operatorname{tg}\gamma_1 + \operatorname{tg}\beta \cos \varphi_2} \\ \operatorname{tg}\tilde{\gamma}_{011} &= \frac{(\dot{\omega}_1/\omega_1^2) - \operatorname{tg}\beta \sin \varphi_2}{1 + (\dot{\omega}_1/\omega_1^2)\operatorname{tg}\beta \sin \varphi_2} & \text{or in the form} & \operatorname{tg}\tilde{\gamma}_{011} = \frac{\operatorname{tg}\gamma_{011} - \operatorname{tg}\beta \sin \varphi_2}{1 + \operatorname{tg}\gamma_{011}\operatorname{tg}\beta \sin \varphi_2}. \end{aligned} \quad (5.3)$$

For the case that $\dot{\omega}_1 = 0, \omega_1 = \text{constant}$

$$\begin{aligned} \operatorname{tg}\tilde{\gamma}_{011} &= \frac{(\dot{\omega}_1/\omega_1^2) - \operatorname{tg}\beta \sin \varphi_2}{1 + (\dot{\omega}_1/\omega_1^2)\operatorname{tg}\beta \sin \varphi_2} = \operatorname{tg}\beta \sin \varphi_2 \\ \operatorname{tg}\tilde{\gamma}_1 &= \frac{1}{\operatorname{tg}\beta \cos \varphi_2} = \operatorname{ctg}\beta \frac{1}{\cos \varphi_2}, \end{aligned} \quad (5.4)$$

where γ_1 is relative angle of rotation in comparison with angle of rotation φ_1 , when $\tilde{\gamma}_1$ is absolute angle of rotor rotation about axis oriented by unit vector \vec{n}_1 , taking into account its rotation about axis oriented by unit vector \vec{n}_2 .

6. Dynamic of Rigid Body Coupled Rotation around Two Orthogonal Axes without Intersection and with One Degree of Freedom

6.1. Model Description of a Gyrorotor Coupled Rotations around Two Orthogonal Axes without Intersection and with One Degree of Freedom

We are going to take into consideration special case of the considered heavy rigid body with coupled rotations about two axes without intersection with one degree of freedom, and in the gravitation field. For this case generalized coordinate φ_2 is independent, and coordinate φ_1 is programmed. In that case, we say that coordinate φ_1 is rheonomic coordinate and system is with kinematical excitation, programmed by forced support rotation by constant angular velocity. When the angular velocity of shaft support axis is constant, $\dot{\varphi}_1 = \omega_1 = \text{constant}$,